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Acoustical Visualization of a Refrigeration Compressor by Using Statistically Optimized Nearfield Acoustical Holography in Cylindrical Coordinates

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ACOUSTICAL VISUALIZATION OF A REFRIGERATION COMPRESSOR BY USING STATISTICALLY OPTIMIZED NEARFIELD ACOUSTICAL HOLOGRAPHY IN CYLINDRICAL COORDINATES

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INTRODUCTION

- NAH is a useful tool for visualizing noise sources throughout a 3D space.
  - Very fast since implementing spatial Fourier transform.
  - Needs zero padding of measurement results to avoid wrap around error.
  - Meaningless velocity results close to measurement edge due to discontinuity.
- Statistically Optimized Nearfield Acoustical Hologrpahy
  - First introduced by Jørgen Hald in planar coordinates
  - NO spatial Fourier transform involved.
  - More accurate result over entire measurement area.

SONAH in Cylindrical Coordinate

- Sound pressure in cylindrical coordinates,

$$p(r,\phi,z) = \sum_{m=-\infty}^{m=\infty} \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{H_m^{(1)}(k_r r)}{H_m^{(1)}(k_r r_m)} P_m(r_h, k_z) e^{im\phi} e^{ik_z z} dk_z$$

where $P_m(r_h, k_z)$ is cylindrical wavenumber spectrum.

- Defining wave function Φ_{Km} in cylindrical coordinates as,

$$\Phi_{\mathrm{Km}} \equiv 2\pi \frac{H_m^{(1)}(k_r r)}{H_m^{(1)}(k_r r_h)} e^{im\phi} e^{ik_z z}, \quad k_r = \begin{cases} \sqrt{k^2 - k_z^{\,2}} & \text{for} \quad |k| \geq |k_z| \\ i\sqrt{k_z^{\,2} - k^2} & \text{for} \quad |k| < |k_z| \end{cases}$$

$$p(r,\phi,z) = \frac{1}{(2\pi)^2} \sum_{m=-\infty}^{m=\infty} \int_{-\infty}^{\infty} P_m(r_h, k_z) \Phi_{\mathrm{Km}} dk_z$$

SONAH in Cylindrical Coordinate

- The sound pressure, $p(\mathbf{r})$, can be expressed as linear combination of the measured sound pressure $p(\mathbf{r}_n)$,

$$p(\mathrm{r}) \approx \sum_{n=1}^{N} c_n(\mathrm{r}) p(\mathrm{r}_n)$$

- If a good representation of the sound field can be obtained by using a finite subset of wave functions, the coefficients c_n can be determined.

$$\Phi_{\mathrm{Km}}(\mathrm{r}) \approx \sum_{n=1}^{N} c_n(\mathrm{r}) \Phi_{\mathrm{Km}}(\mathrm{r}_n), \qquad m = 1 \ \ldots \ M$$

SONAH in Cylindrical Coordinate

- Rewriting the quantities in the form of matrices and vectors,

$$\mathrm{A} \equiv \left[\Phi_{\mathrm{Km}}(\mathrm{r}_{\mathrm{n}})\right], \quad \alpha(\mathbf{r}) \equiv \left[\Phi_{\mathrm{Km}}(\mathbf{r})\right], \quad \mathrm{c}(\mathbf{r}) \equiv \left[c_n(\mathbf{r})\right]$$

- Finite subset of wave functions can be written by using matrices and vectors,

$$\alpha(\mathbf{r}) \approx \mathrm{Ac}(\mathbf{r})$$

- Regularized least square solution **c**(**r**) is,

$$\mathrm{c}(\mathbf{r}) = \left(\mathrm{A}^{+}\mathrm{A} + \theta^{2}\mathrm{I}\right)^{-1} \mathrm{A}^{+}\alpha(\mathbf{r})$$

where, regularization parameter θ is,

$$\theta^{2} = \left[\mathrm{A}^{+}\mathrm{A}\right]_{nn} 10^{-SNR/10}, \quad \text{e.g., SNR= 40 dB}$$

SONAH in Cylindrical Coordinate

- Estimated pressure $p(\mathbf{r})$ is,

$$p(\mathbf{r}) \approx \sum_{n=1}^{N} c_n(\mathbf{r})p(\mathbf{r}_n) = \mathbf{p}^T\mathbf{c}(\mathbf{r}) = \mathbf{p}^T\left(\mathbf{A}^+\mathbf{A} + \theta^2\mathbf{I}\right)^{-1}\mathbf{A}^+\boldsymbol{\alpha}(\mathbf{r})$$

where, $\mathbf{p}^T$ is measured pressure vector at $\mathbf{r}_n$

- Estimated radial particle velocity $u_r(\mathbf{r})$ is,

$$u_r(\mathbf{r}) \approx \mathbf{p}^T\left(\mathbf{A}^+\mathbf{A} + \theta^2\mathbf{I}\right)^{-1}\mathbf{A}^+\boldsymbol{\beta}(\mathbf{r})$$

where, $\mathbf{A}^+\boldsymbol{\beta}(\mathbf{r})$ is a correlation vector that relate measured pressure and particle velocity.

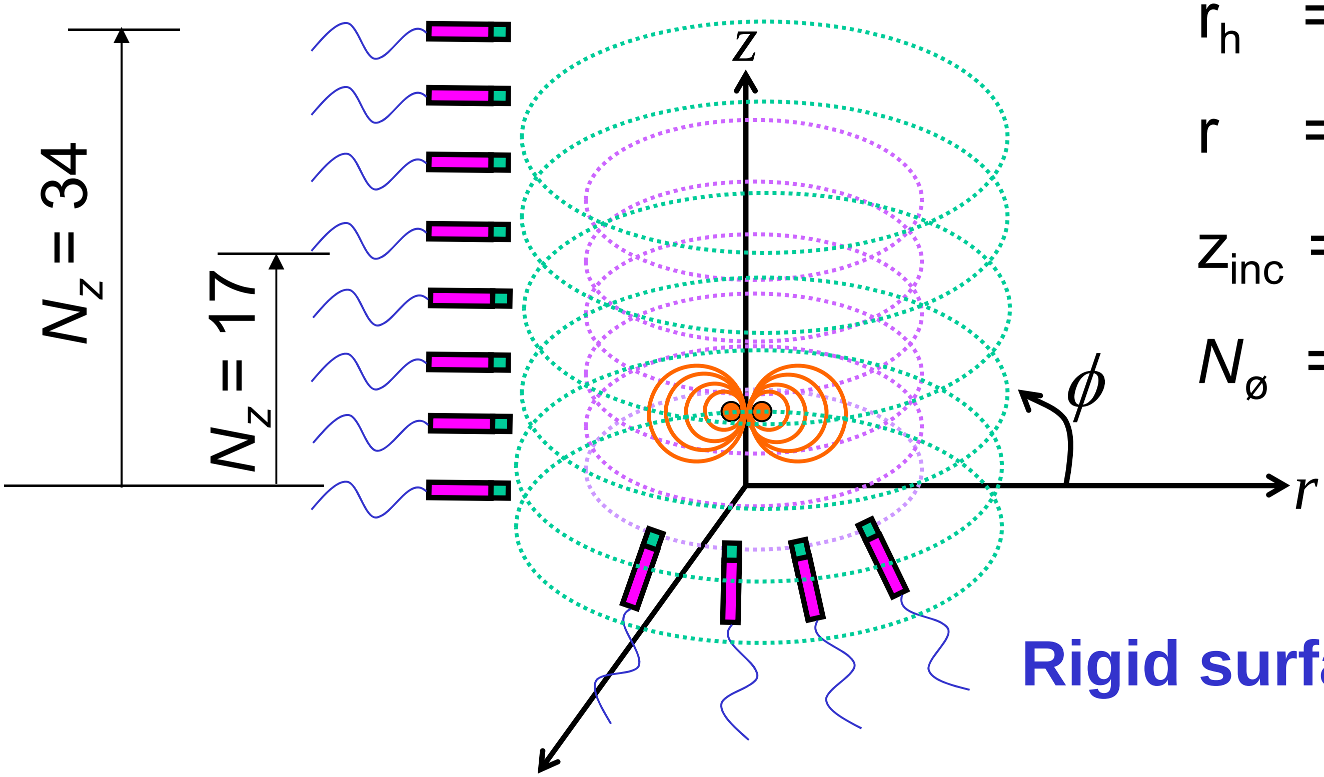
SONAH in Cylindrical Coordinate

- To estimate $N_\phi N_z$ *by* $N_\phi N_z$ square matrices $\left[\mathrm{A}^{+}\mathrm{A}\right]_n$, $\left[\mathrm{A}^{+}\alpha\right]_n$, $\left[\mathrm{A}^{+}\beta\right]_n$ more effectively, avoid repeated calculations as much as possible.

$$\begin{bmatrix}
a_{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{2,1} & a_{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{3,1} & a_{2,1} & a_{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{4,1} & a_{3,1} & a_{2,1} & a_{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{33,1} & \cdot & \cdot & \cdot & \cdot & a_{1,1} & \cdot & \cdot & \cdot & \cdot \\
a_{34,1} & a_{33,1} & \cdot & \cdot & \cdot & a_{2,1} & a_{1,1} & \cdot & \cdot & \cdot \\
a_{35,1} & a_{34,1} & a_{33,1} & \cdot & \cdot & a_{3,1} & a_{2,1} & a_{1,1} & \cdot & \cdot \\
a_{36,1} & a_{35,1} & a_{34,1} & a_{33,1} & \cdot & a_{4,1} & a_{3,1} & a_{2,1} & a_{1,1} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{bmatrix}$$

Dipole Numerical Simulation

- Dipole axes in ø=0, z=5 cm and ø= 90º, z=25 cm



r_h = 14.15 cm

r = 9.0 cm

z_{inc} = 2 cm

$N_ø$ = 32

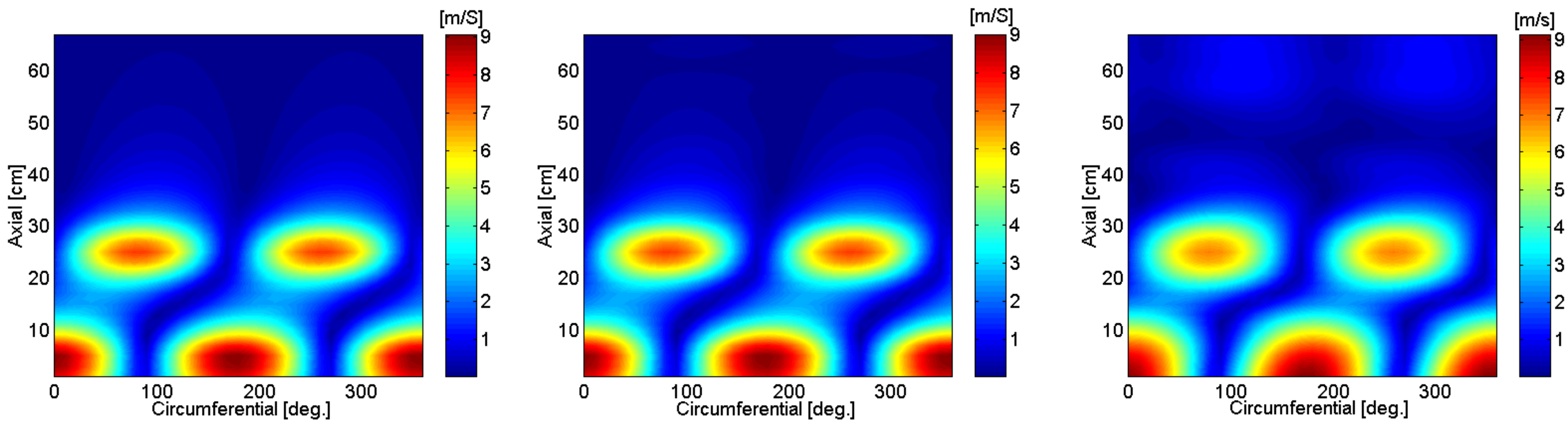
Rigid surface at *z*=0

Dipole Numerical Simulation

Directly measured and backward projected particle velocity

(N_z = 34)

Directly measured | SONAH (MSE : 1.3 %) | NAH (MSE : 14.9 %)

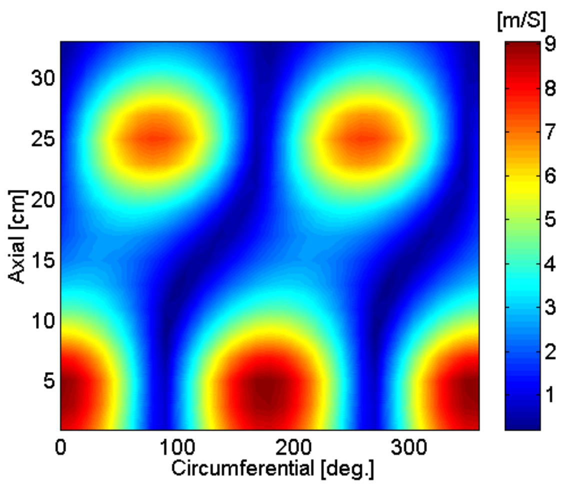


Dipole Numerical Simulation

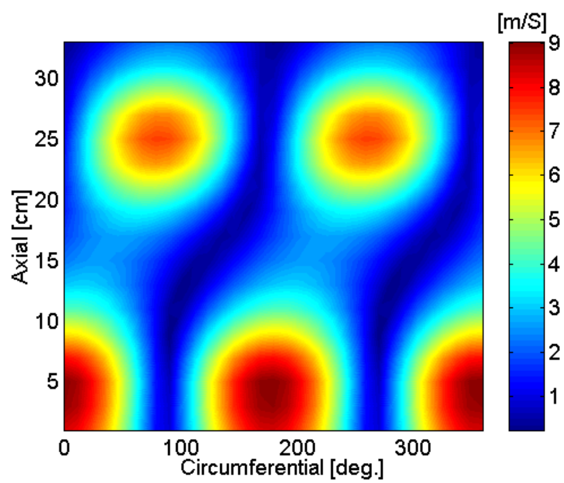
Directly measured and backward projected particle velocity

(N_z = 17)

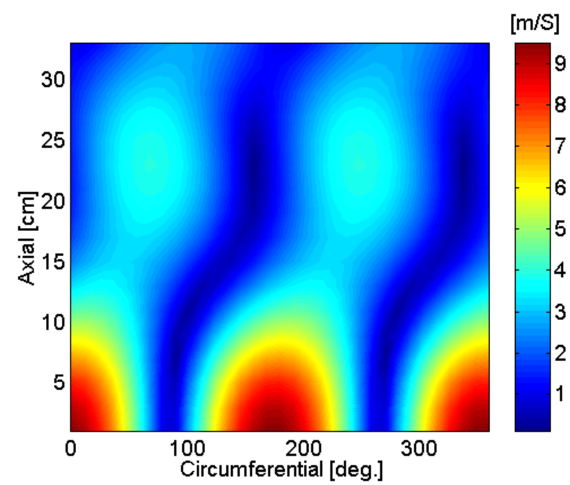
Directly measured



SONAH
(MSE : 1.6 %)

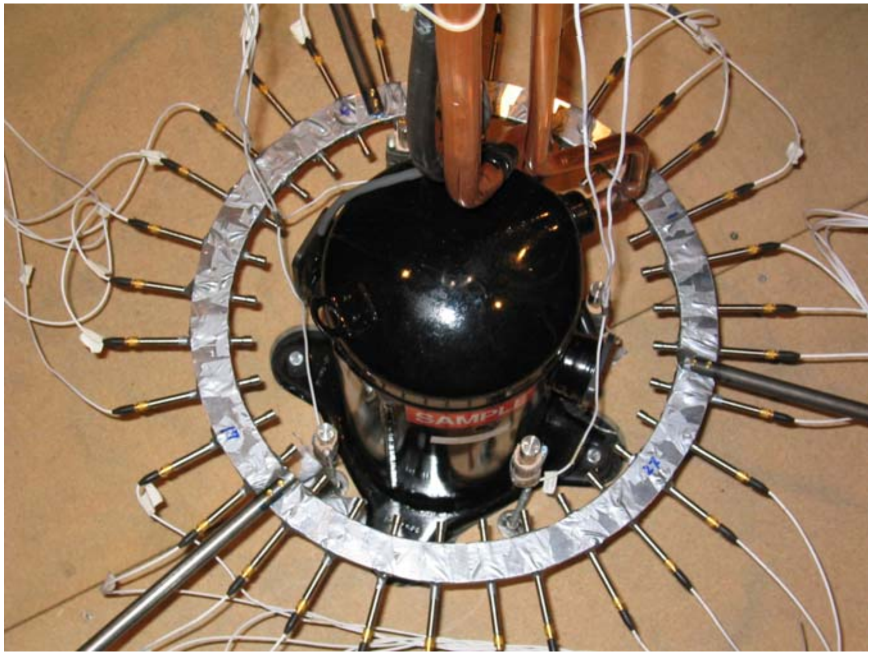


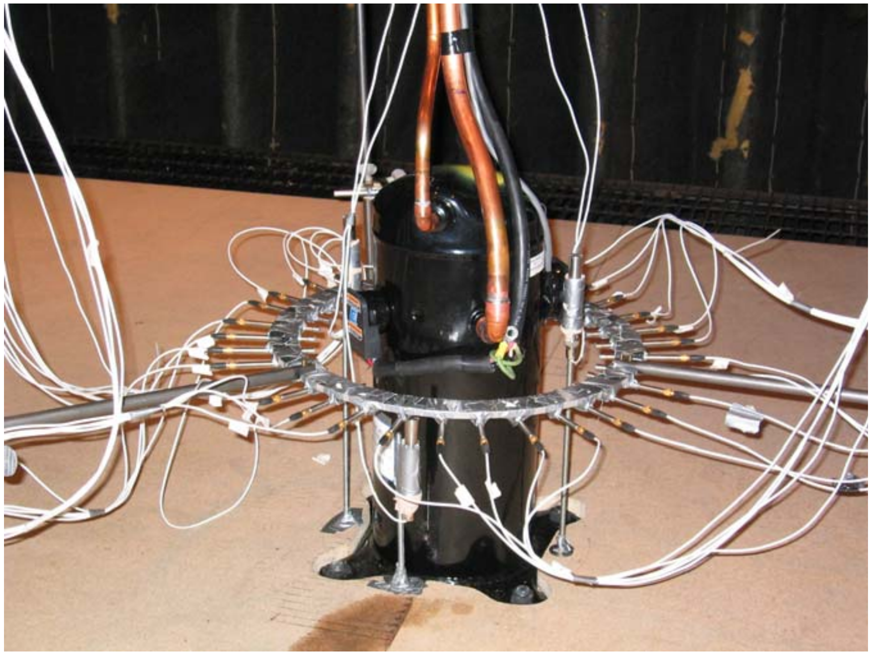
NAH
(MSE : 31.6 %)



Compressor Measurement

- Number of field microphones : $N_\phi = 32$
- Microphone spacing in *z* direction : $z_{inc} = 2$ cm
- Radius of hologram : $r_h = 14.15$ cm
- Total aperture size : 67 cm (N_z=34)



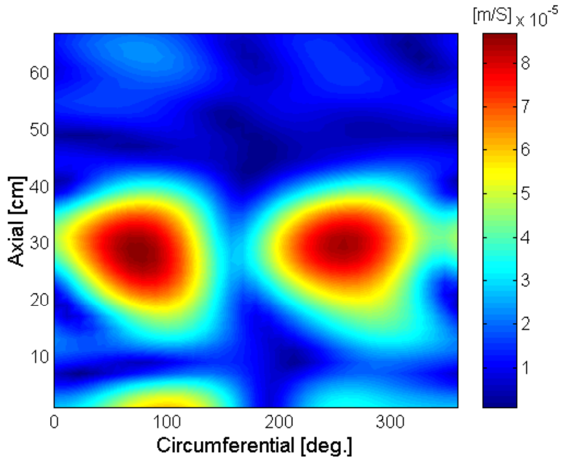


Compressor Measurement

Backward projected velocity (882 Hz)

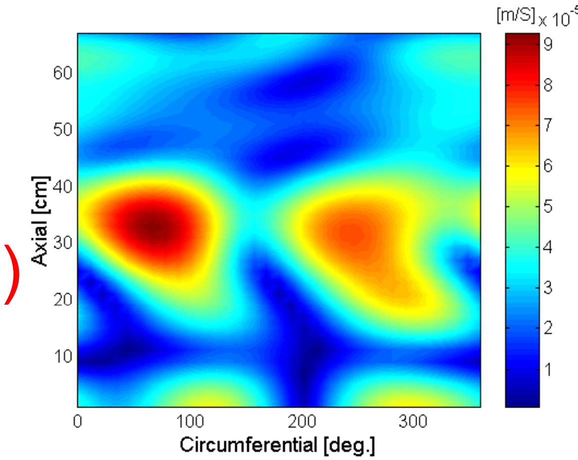
NAH

$(N_z = 34)$



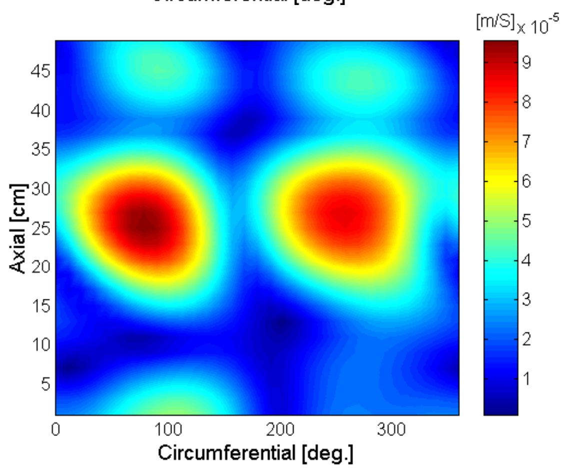
SONAH

$(N_z = 34)$



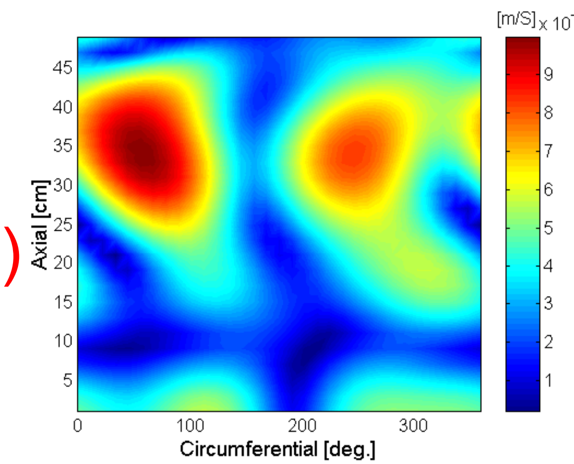
NAH

$(N_z = 25)$



SONAH

$(N_z = 25)$

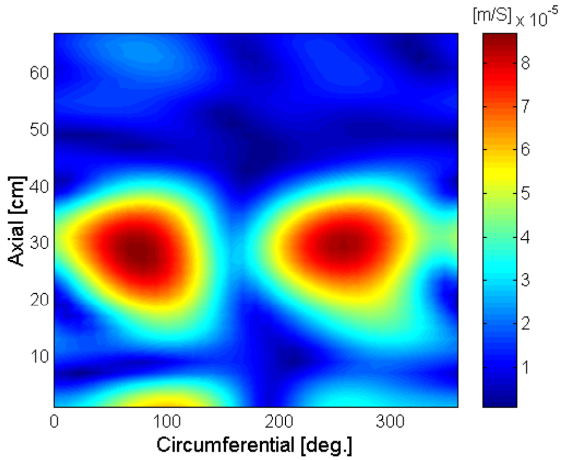


Compressor Measurement

Backward projected velocity (882 Hz)

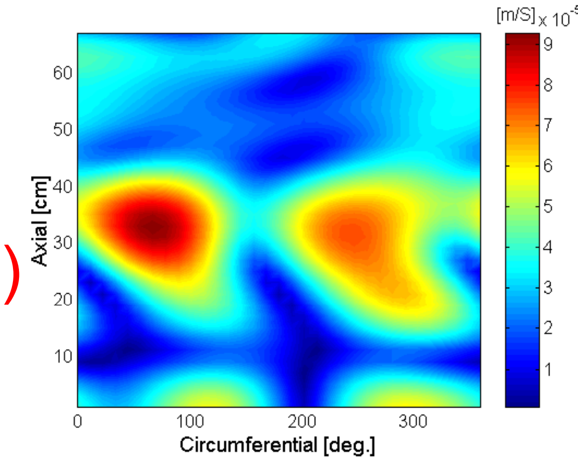
NAH

(N_z = 34)



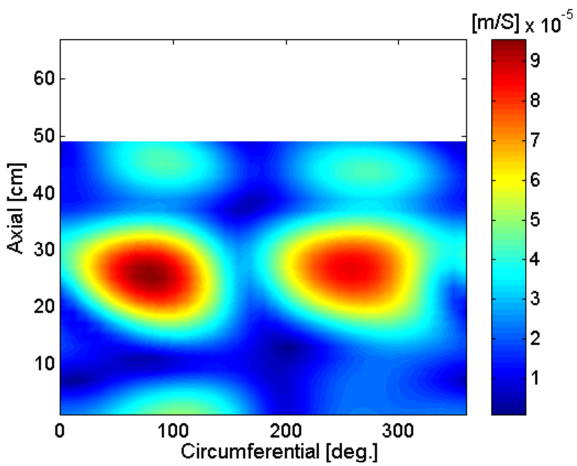
SONAH

(N_z = 34)



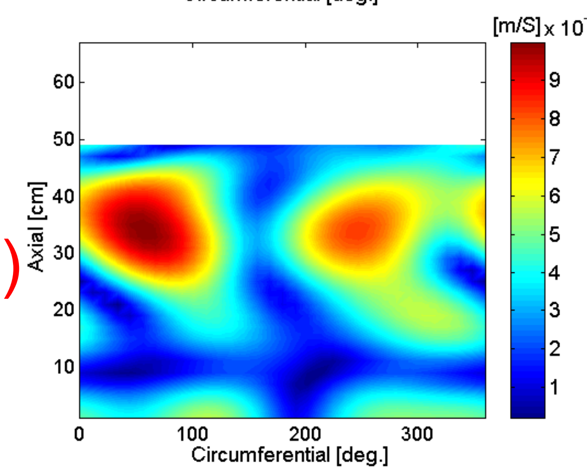
NAH

(N_z = 25)



SONAH

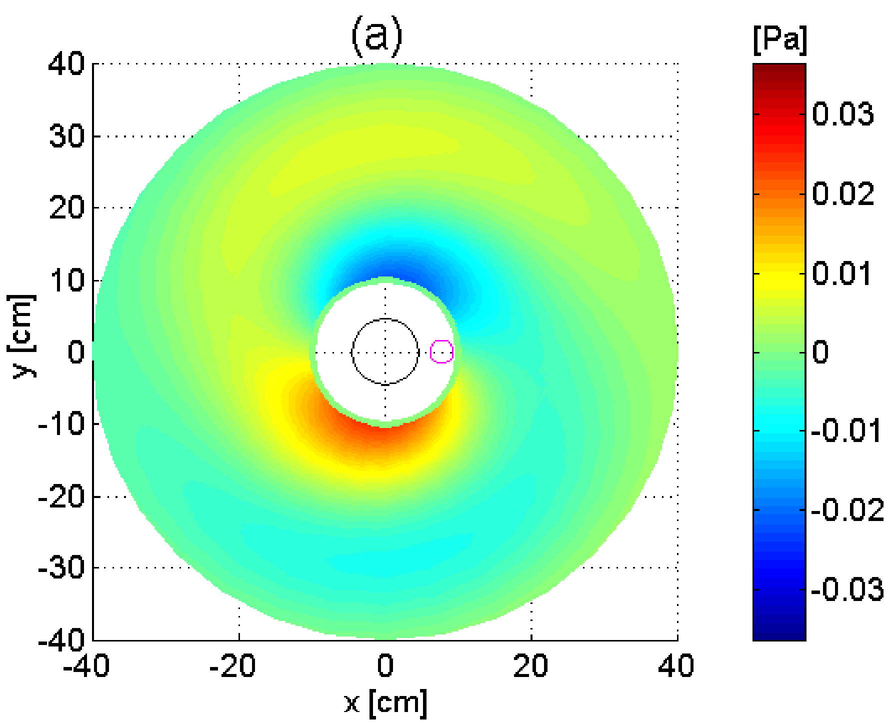
(N_z = 25)



Compressor Measurement

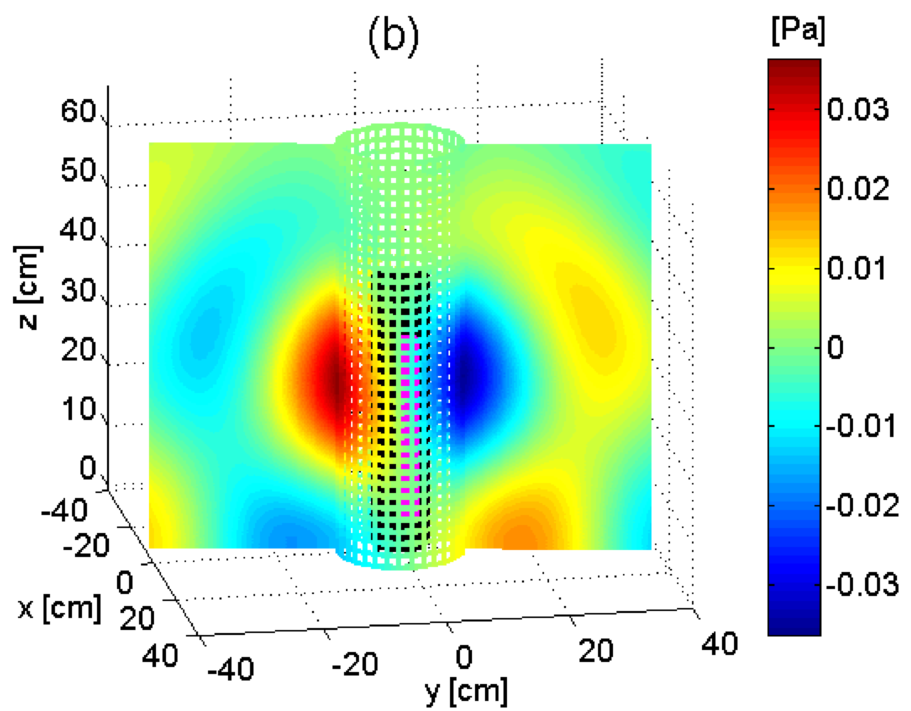
Time domain visualization

(Horizontal, 882 Hz)



Time domain visualization

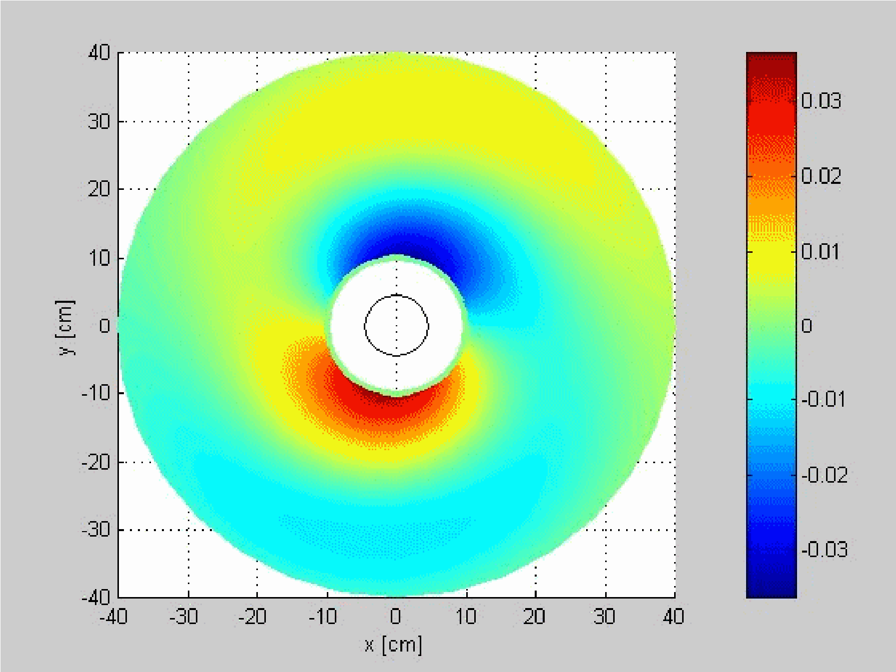
(Vertical, 882 Hz)



Compressor Measurement

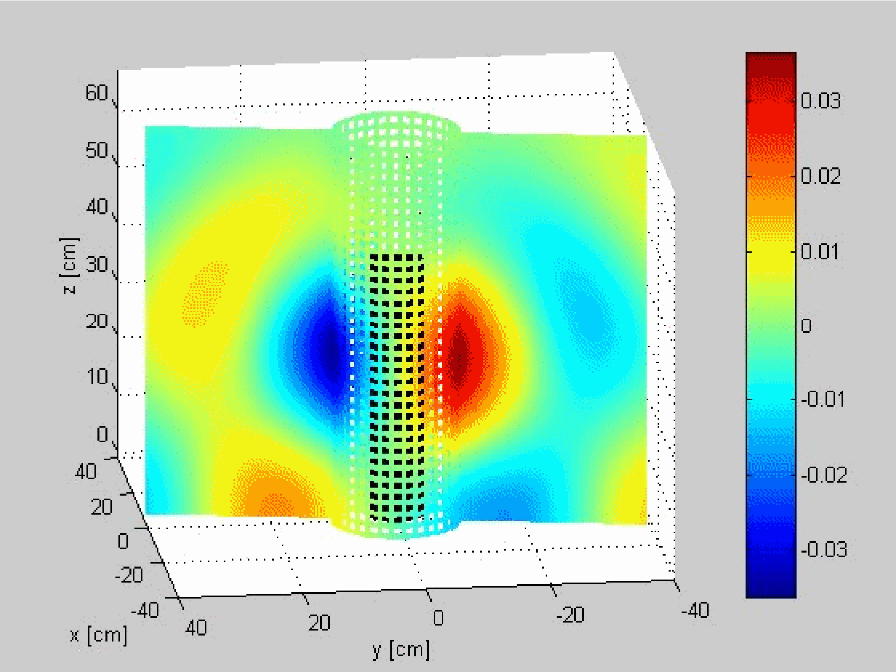
Time domain visualization

(Horizontal, 882 Hz)



Time domain visualization

(Vertical, 882 Hz)



Conclusions

- SONAH procedure was implemented in cylindrical coordinates, consistent backward projections results were obtained even when the measurement aperture size was decreased.
- By avoiding repeated calculation, the SONAH calculation time can be reduced dramatically, which makes it practical to use SONAH if it is not possible to make measurements in the region where the sound pressure drops to negligible levels.